Exam. Code : 103201
Subject Code : 1025

B.A./B.Sc. 1st Semester (Batch 2021-24)

MATHEMATICS

Paper—I

(Algebra)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt *five* questions in all, selecting at least ***one*** question from each section. The **fifth** question may be attempted from any section. All questions carry equal marks.

SECTION—A

1. (a) Prove that system of vectors u = (1,2,–3); v = (1,–3,2) and w = (2,–1,5) of $V_3(R)$ is L.I.

   (b) Show that the vector v_1 = (2,0,0,); v_2 = (0,3,0); and v_3 = (0,0,4) are linearly independent.

2. (a) Define rank of matrix and nullity of matrix with examples.

   (b) Determine the rank of the matrix A given below :

$$A = \begin{bmatrix} 3 & 3 & 1 \\ 5 & 3 & 2 \\ 1 & 2 & -1 \end{bmatrix}$$

SECTION—B

3. (a) Find the eigenvalues and eigenvectors of this 3 by 3 matrix A :

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

(b) Find the eigenvalues and eigenvectors of the following matrices :

$$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & 4 \\ 2 & 4 \end{bmatrix}$$

4. State and prove Cayley Hamilton theorem.

SECTION—C

5. (a) Prove that the rank of values of two congruent quadratic forms is the same.

(b) Reduce to diagonal matrix by congruent transformations :

$$A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 0 & 3 \\ -1 & 3 & 1 \end{bmatrix}$$

6. (a) Prove that if A be any skew-symmetric matrix over F, there exist a non-singular matrix P over F such that :

$$P'AP = \text{diag}[J, J ..., J, 0, 0, ..., 0],$$

where

$$J = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

(b) Prove that, the form X'AX has the value 0, if and only if X = 0.

SECTION—D

7. (a) If a, b, c are the roots of the equation $x^3+p_1 x^2+p_2 x+p_3 = 0$, form the equation whose roots are s^2, b^2, c^2.

(b) Explain the Descartes' Rule of signs.

8. (a) Find the Cardan's Solution of equation $x^3+qx+r = 0$

(b) Solve the equation $x^4-2x^2+8x-3 = 0$ by Descarte's method.